Compaction / extension - the combination for optimum learningThe Mathematically Gifted - Different Thinking May Require Different Teaching

(a version of this paper, titled Killing Time, was published in Principal Matters, April, 2001)

Lynne Kelly, BE (Elec.), Dip. Ed., Grad Dip. Comp., M. Ed (CHIP); Principal, Virtual School for the Gifted; Coordinator, Gifted and Talented Program, Methodist Ladies' College, Melbourne, Victoria 3101

Professor Michael W. O'Boyle, Ph.D., Morgan Chair of Psychology and Director of the Morgan Centre for the Study of the Development of High Intellectual Potential, School of Behavioural Science, University of Melbourne, Victoria 3010

Do mathematically able students just learn their mathematics faster than average students? Or, is there something fundamentally different in the way they mentally process mathematical concepts and approach problems? If there is a fundamental difference, what are the implications for teaching our most able maths students?

While the debate over who is actually gifted continues, suffice it to say that in this paper we are talking about children who are obviously gifted in mathematics. There is, however, no such thing as a typical gifted child - they do not form a homogenous mob. Children may be gifted in some subject areas and not in others. The asynchrony in their development and interest needs to be built into their curriculum options. Gifted students are capable of more than just the same stuff presented at a faster rate. Research now indicates they need exposure to qualitatively different material that enhances their particular abilities and gifts, and capitalises on the manner in which they tend to process new information. Gifted students are known to think differently and may rely upon different brain structures and circuitry when doing so (O'Boyle, 2000a; 2000b).

In over a decade's work focusing on the functional organisation of the gifted brain, O'Boyle, et al (1995) have suggested that the mathematically gifted brain is qualitatively (not just quantitatively) different from those of their average ability cohort. And, using a variety of neuropsychological measures, they have shown that enhanced development of the right cerebral hemisphere and an unusual reliance upon it when processing information for the purposes of learning, may be key characteristics of the mathematically gifted brain. In addition, the mathematically gifted exhibit an enhanced ability to switch and adjust brain activation levels in an orchestrated and co-ordinated manner between the left and right hemispheres when processing new information. This may be accomplished via a brain structure known as the Corpus Callosum - the major connecting fibre between the two hemispheres. Thus, the ability to optimise the use of both hemispheres may be a special aspect of gifted brain functioning. Interestingly, other researchers (Witelson, et al., 1999) have examined the brain of Albert Einstein and found it to be characterised by enhanced development of the parietal lobes, particularly on the right side. This is an area known to be related to the formation of mental images and is the very area that has been implicated in the O'Boyle, et al (1995) research as being related to mathematical precocity in adolescents.

In this light, it is interesting to reflect on what Einstein wrote in a letter to Jacques Hadamard and quoted in the classic work The Mathematician's Mind (Hadamard, 1996, originally published in 1945)

"The words of the language, as they are written or spoken, do not seem to play any role in my mechanism of thought. The psychical entities which seem to serve as elements in thought are certain signs and more or less clear images which can be 'voluntarily' reproduced and combined.... The above mentioned elements are, in my case, of visual and some of muscular type. Conventional words or other signs have to be sought for laboriously only in a secondary stage, … "

In Our Classrooms

What are the implications of this new way of considering the qualitatively different mechanisms that seem to underlie gifted maths ability? How can we translate this into the classroom? Consider the cases of four hypothetical students.

The imaginary students used in this discussion are represented on the curve above. These scores reflect each student's level of performance in mathematics on an arbitrary scale; they do not represent their relative abilities in other subjects.

Sandy: An average kid in the class. The regular curriculum is designed for Sandy.
Tam: A bright kid. Maybe defined as gifted, maybe not. Tam does well in mathematics.
Chris: A really bright kid. Chris is capable of really standing out in mathematics.
Kim: Is so highly able that the general curriculum holds little if any challenge. It may be that Kim's extreme abilities are in a narrow area, maybe more so in maths than other subjects. The more extreme the ability the more individualised the program needed.

Any of these students may be performing below their ability level. Giftedness is a potential and there are many reasons for under-achievement. For example, boredom can be a major factor. So in any given class you may have 28 students with lots of Sandys, a few Tams, maybe a Chris or two, and occasionally a Kim. There are probably a few in the class needing learning support as well. All have equal rights to learn - not just sit in the class, but to actually learn to their potential.

If a student can demonstrate complete mastery of a topic there is little reason to spend additional time studying it. They might do none of the topic, or only a small part - whatever they need to score highly (but not perfectly) on the final assessment. It is essential that assessment is not busy work such as a long research project where factual information is shifted from source to report, with only the wording changed to make it "original".

Teachers need to enrich the student's ability to explain alternative methods, and not to force methods upon them that are designed for the majority. The gifted think, process and feel differently about maths and hence engage concepts and components in a different manner, and thus require different modes of expressing their ability.

Classroom Realities

Teachers are often tired and usually flat out just covering the standard syllabus. Managing a compaction and enrichment model in the classroom is a significant ask of any teacher. The key is to realise that any given teacher will be virtually fully occupied with the rest of the class. Thus "teaching" another more advanced topic is not a realistic expectation. It is probably not be in the best interest of the class to be moving on to topics that are well beyond the regular curriculum. It is far better to be using the ability of gifted students to think qualitatively differently and bring into the extended curriculum, material that cannot be readily completed by the average student.

For example, a little test: as a teacher, can you think of topics that excite you in your subject area, but are glad you do not have to (or could not) explain to the whole mixed ability class? Is there an instinct you are aware of for students who just feel a particular subject area? You may not be able to define it, but you can often recognise it. That is the foundation of the qualitatively different curriculum of which we speak.

The task then is to recognise the different learning styles of the extremely able and cater to them via selective extension tasks. While Sandy will ask, "how do I do this problem?" and apply the methods taught, as we move up through Tam, Chris and Kim, the approach may be very different. As we get to Kim, there is an instinct which sees the mathematics, feels it and gets answers intuitively, often unable to quote a method unless that experience has been part of their enrichment expectations. This scenario, of course, assumes the problems have sufficient difficulty to challenge Kim, which is not always true in the average classroom.

Highly gifted maths students are not always first in tables races. Nor do they finish simple arithmetic tasks extremely quickly, or even accurately. Ability at mental arithmetic, while a sign of intelligence, may be more a sign of good memory than mathematical instinct. Many mathematically able individuals have average memories, but are strong in logic. Hence, give them 6 x 8 and they may remember 6 x 7 and add another 6. It is a slower process, but correct. Thus, memory and arithmetic ability are not reliable indicators of exceptional maths ability.

From a very young age teachers need to ask high ability maths students to explain their reasoning. Talk mathematics. Feel mathematics. Play with mathematics. Enjoy it in their own way, without the pressure to conform to standard methods. They need to learn how to communicate their thinking. Teachers must be willing to accept variation in the way mathematical solutions are presented. As long as it is communicated clearly and is mathematically correct, does it matter if it does not match the exact method taught to the class - a method often geared to the average ability majority?

An Example to Try

One clear sign of mathematical ability is the tendency to engage a given problem, and extend beyond it. This can not be done in time-limited situations. Consider a simple example problem that seems to differentiate between maths ability levels. Note that before beginning the task, the number FOUR is written elsewhere.

Accurate Prediction

I have written down a number elsewhere, and I am sure to predict the outcome to your calculation. I am able to control your mind to ensure you write down lettering, which will work. Or am I?

I want you to write down your name - your whole name. Spaces, middle name, whatever you like. I'm sure to be right whatever you write.

Count how many letters, spaces and any special characters like apostrophes or hyphens. Write down - in words - the number you counted.
Count the number of letters, hyphens and spaces in the word or words for the number. Write it down in words.
Count the number of letters, hyphens and spaces in the word or words for the new number. Write it down in words.

Keep doing this until you keep getting the same result over and over. The "keep doing this bit" is called Iteration. Iteration is taking the result of one process and starting with it to do the process over again. You then take that result and do it again, and again, and again. Iteration is a very commonly used process. In fact Nature is just one long iterative process. But that's another story!

Have you come to the point where you just get the same thing over and over? Let's see if I predicted it right. (Check against the previously written number FOUR.)

Why was I right?
How do you know? Can you explain why I will always be right?

Informal observations over many trials of this problem have indicated that Sandys will try a few different examples and say, "yes, it always works." Some will say "so what?" The Tams will probably look at the words and note something about FIVE, NINE and FOUR as always ending up at FOUR, and hence discover a processing loop. Tam may tell you how the larger numbers all collapse to the smaller ones, and Chris will probably give an exhaustive proof, showing that each number must end up in the smaller numbers, and how the loop works for every possibility, and thus prove they must all end at FOUR. Chris will then probably note that this is dependent on the English language and try some others noting that different languages lead to different processing loops and different endpoints.

Kim (and maybe Chris) will usually move to a generalisation. This tendency to try to generalise and construct absolute rules is yet another sign of high maths ability. Kim will probably see the relationship to iteration and other behaviours that are iterative. For example, Kim might link this processing loop to science fiction time travel or some related phenomenon. Or, Kim might try to construct a language where there is no loop or fixed endpoint. Given the opportunity, Chris or Kim might describe numbers as images of light bouncing around as crystals in their head. The time spent on the problem varies inversely with the mathematical ability. And, faster is rarely better.

Interestingly, an older Sandy does not behave like a younger Kim. This is the secret of qualitatively different material. It generally will not engage an average student of any age. It does not involve a method that can be taught or applied without understanding. It requires the instinctive engagement of a qualitatively different way of thinking and feeling about mathematics.

So how can such extensions be managed in the mixed ability classroom, where the other twenty-four students are in need of basic maths instruction? One method, described below, not only handles classroom management, but more importantly, is educationally valid as well.

Dear Teacher, … cheers, Student.

Here is a suggestion - have gifted students send their teacher letters or emails. Dear Ms … or Hi Mr … They do not demand time during class, amid all the other demands, and refusal of the teacher for instant gratification of the correct answer ensures the opportunity for the student to reflect and extend. There is no closure of the usual tick, but there is also no reward for avoiding risks.

The reason the above method seems to work is that students never write "Dear Teacher, 23. Luv Chris." Or "Dear Mrs Educator, 1a. 35, 1b. negative. Regards, Tam." They feel obliged to say something. It is a letter after all. They always say more which immediately enables the teacher to probe for further expression of their thinking. The teacher does not grade their letter, they write back, asking for clarification. They suggest alternatives. They praise the proof that did not work, and the generalisation that is almost there. They can rejoice in the pattern found in primes that did not always hold up, but represented a good try anyway.

Through these letters students communicate their thinking, solutions, ideas, hassles, what the dog did, silly mistakes, news of a broken arm and anything else which comes to mind. More importantly, teachers want the students to take time to reflect and problem pose, to engage and link to other disciplines, to generalise and try for a proof. Teachers want their students to verbalise their mathematical thinking, reflect upon it, and take time to appreciate and develop their toolbox of techniques. Teachers want them to envision other methods they might use to gain a deeper understanding of their metacognitive approach to maths. This is what compaction allows them to do. Rush the easy stuff and indulge the higher level skills.

In terms of formal assessment, when the teacher asks for working to be shown on a maths test, most gifted students have the skills to explain their own working, and most teachers are happy to accept that. It is the lack of working and explanation that is judged to be unacceptable. Most good maths teachers accept a well reasoned alternative method. There is a balance between allowing and encouraging instinctive methods and the need for students to be able to communicate their thinking. Materials and methods that encourage qualitatively different thinking are now beginning to optimise the learning of mathematically gifted students and their ability to think and communicate about mathematics.

What is Qualitatively Different Mathematics?

There are many references in the literature (sometimes taken from biographies and interviews of highly able maths students) which describe the type of problems teachers should be offering to encourage their qualitatively different way of thinking. These are typically problems that are not accessible to the average student and are generally not included in the regular curriculum to any depth. Topics such as game theory, number theory, non-Euclidean geometry, logic puzzles, paradoxes, fractal geometry and so on, are examples. As are most problems which involve patterns and images, and that are open and have significance and relevance, things which enable students to feel the aesthetic side of mathematics. Teachers need to rely on more than just the great problems (Mobius Strip, Hotel Infinity, Codes and Ciphers, Magic Squares, problems on primes and irrationality, etc.), they need to include what theorists in gifted education refer to as scope and sequence. There need to be measurable educational outcomes, a sequential development of skills, curriculum goals and so forth. School should not be a matter of "killing time" but rather a place to nurture these students' advanced instincts and feelings about maths.

Mathematically gifted students can do more than just cover the regular curriculum faster and better. They are capable, to varying degrees, of qualitatively different kinds of thinking, completing more complex tasks at each level, and expressing a feeling and instinct for all things mathematical. As teachers, it should be our mission to create a classroom environment that caters to these special needs.

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